Three applications of AM-GM Inequality.

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Prove inequalities

(**a**) $(1+x)^n \le \dfrac{1}{1-nx}$ for $x \in [-1,0], n \in \mathbb{N}$.

(**b**) $(1+x)^r \le 1 + \dfrac{rx}{1-(r-1)x}$ for $x \in \left[-1, \dfrac{1}{r-1}\right), r > 1$.

(**c**) $(1+nx)^{n+1} \ge (1+(n+1)x)^n$ for $x \in \mathbb{R}_+, n \in \mathbb{N}$.

Solution by Arkady Alt, **San Jose**, **California**, **USA**.

(**a**) By AM-GM Inequality $((1+x)^n(1-nx))^{1/(n+1)} \le \dfrac{n(1+x)+1-nx}{n+1} = 1$.

Hence, $(1+x)^n(1-nx) \le 1 \Leftrightarrow (1+x)^n \le \dfrac{1}{1-nx}$.

(**b**) By AM-GM Inequality

$$\left((1+x)^{r-1} \cdot (1-(r-1)x)\right)^{1/r} \le \frac{(1+x)(r-1)+(1-(r-1)x)}{r} = 1.$$

Hence, $(1+x)^{r-1} \le \dfrac{1}{1-(r-1)x} \Leftrightarrow (1+x)^r \le \dfrac{1+x}{1-(r-1)x} = 1 + \dfrac{rx}{1-(r-1)x}$.

(**c**) By AM-GM Inequality $(1+(n+1)x)^{n/(n+1)} = (1 \cdot (1+(n+1)x)^n)^{1/(n+1)} \le \dfrac{1+n(1+(n+1)x)}{n+1} = 1+nx \Leftrightarrow (1+nx)^{n+1} \ge (1+(n+1)x)^n$.